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ON THE NATURE OF SCIENTIFIC LAW AND SCIENTIFIC EXPLANATION.¹

THE search for the occult causes, the hidden springs, that move the machinery of that great and unending panorama of sensuous phenomena which we call the universe, is as old as the human mind itself. The explanatory principles of the world, the elements from which it was supposed to have been constructed, were called by the Greeks *ἀρχαί*, which means the beginnings, the ultimate elements. These architectonic elements, the primordial mosaics and building-blocks of every world-view, took their distinctive form and coloring from the type of individual and national mind, the stage of civilisation, the dominant characteristics of the thought and occupation of the period. With the savage they were spiritual and animistic; with the Hindus they were mystical and ultra-esoteric; with the Jews, ethical and theological; with the Phœnicians, if anything, commercial; with the Romans, political and juristic; with the Greeks, more abstract and in every way more symmetrical: the cobbler cannot eliminate his last from his world-view; the tailor his goose. They all represented abstractions, more or less partial and incomplete, of the features of reality; they were rarely com-

¹ The present paper was read before the Physical Section of the Science Club of Northwestern University at Evanston, in February, 1899, and in part also before the Chicago branch of the Esoteric Extension, in December, 1898. This latter occasion will account for the frequent references to mysticism. The paper does not profess to be a discussion of all the aspects of the intricate problem indicated by its title, but is intended rather to emphasise and elucidate some of the more salient points of view that have issued from relatively recent research in the philosophy of science. On this point and in order to dispense with detailed references in each case, the author wishes to acknowledge at the outset his general indebtedness to the works of Dr. Paul Carus, and especially to those of Prof. Ernst Mach of Vienna.

prehensive; they rarely embraced the essential and the universal. But they all bore the same distinctive trait; they were an effort for mental economy,—an attempt to reconstruct the Many from the One, to fix the incessant and dizzying flux of events by a few simple and familiar concepts: they were all more or less monistic.

To trace the development of all these varied struggles of the human intellect to compass Being, from their initial crudity to the refinements of modern monism, is the business of the history of philosophy and science; we are concerned here with the central and linking concepts only of these attempts at explaining the world,—the concept of natural law and the nature of scientific explanation generally.

DEVELOPMENT OF THE NOTION OF LAW. PYTHAGOREANISM, AND THE SCIENCE OF THE GREEKS.

Psychologically speaking, it would be a mistake to suppose that the concepts of natural causality and natural law, in their modern scientific acceptation, are parcel of the primitive and necessary furnishing of the mind. Like all other things in nature, they too are products of evolution.¹ They have grown and constituted themselves in the human brain from the very simplest beginnings, and have passed through all the demonstrable stages of development of that magnificent and multifarious commonwealth of feelings, images, and concepts, which we call the human soul. Starting with mere sense-impressions of regular sequences or of periodicities (as the succession of day and night, the recurrence of the seasons, etc.), and taking initial shape merely as vague and subconscious sentiments of happenings-before and happenings-after,—being more in the nature of expectations than convictions: then, by the continuing incessant action of external nature and the responsive *reflective* action of internal nature, the mind itself, gradually assuming the aspect of a composite photograph, with everything blurred but the essential features; and finally by a supreme and absolute effort of abstraction stripping themselves of every

¹ Ribot, *Evolution of General Ideas*, pp. 180 et seq.

vestige of their concrete forms :—these concepts of natural causality and natural law have, in the world in which we have taken our being, evolved along with us through every ascending stage from the phase in which they led the geese of an English village to the town-square on market day, in the expectation of finding the spilt grains of corn,¹ to the magnificent outbursts of determinism which characterised the utterances and the achievements of a Newton and a Laplace, and the conceptions of a D'Alembert and a Kirchhoff.

Even in historical times the transformations wrought in these concepts have been nothing short of marvellous. The ideas of natural law and of causality which obtained in the seventeenth and eighteenth centuries differed totally from the ideas held by antiquity on this subject, and differ greatly from the ideas held by scientists to-day. To the Romans, the order of nature, like the order of their empire, was entirely social and juristic; their conceptions of law and causality were based upon their own most immediate experience,—that of a police force; and it was this conception that was given to the Christian science and the Christian philosophy of the Middle Ages. But mingled with it was the horrible legacy from the East,—a legacy of portents, miracles, catastrophies, monsters, and freaks, which made of Nature a veritable pandemonium of disconnected events and erratic happenings. Such is the picture portrayed in the companion-pieces to the Revelation of St. John that sprang up mushroom-like in Asia Minor in the first centuries of our era; a picture in which every tribal deity and demiurge shaped Nature to conform to the demands of his own special prophets.

But in all this vast expanse of Stygian darkness was one bright spot, which had its inception in the dimmest epochs of the past, which shone with intense brilliancy for a few ages, then flickered and went out, till it was relighted in the sixteenth century. I refer to the philosophy and science of Greece. Here the conception of the universe as a law-determined cosmos took its being, and its

¹ Romanes, *Animal Intelligence*, p. 314.

rise was concomitant with the development of Grecian Mathematics.

The giant personality in that development, as we are considering it, was the mystical figure of Pythagoras. It was reserved for him to make a really decisive epoch in the history of science and philosophy. Where others had grasped only the fleeting and unessential features of existence he seized the eternal, the universal, and the necessary. He was the first really to appreciate the rôle which the laws of form played in any philosophical scheme of explaining the world. He saw in mathematics, in which he was a great discoverer, the very backbone of existence, the core which ran through the universe, the support and essence, as it were, of Being. So successful was he in resolving reality in the crucible of numbers and geometric relations,—we know his explanation of harmony and of the motions of the planetary bodies,—that the entire science became permeated for him with a divine afflatus, and the secret doctrine, the genuine esotericism of truth, to which only the pure were admitted, was made synonymous with mathematics: the *ἐσωτερικοί* in all the subsequent history of Pythagoreanism were the *μαθηματικοί*.

It was indifferent that his doctrine degenerated into a numerical mysticism. He had struck the keynote which resounded through every philosophic and scientific symphony of the ages to come. He had disentangled the red thread of the universe which the human mind was ever afterward to follow.

To analyse the reasons for the general failure of the Greeks in the experimental sciences, is not a part of our main task. Leucippus stated the atomic theory as clearly as any modern physicist; Democritus formulated the doctrine of the specific energies of the nervous system almost as distinctly as Johannes Mueller; Anaximander anticipated Kant and Laplace in cosmogony, Lamarck and Darwin in biology. Yet these speculations lacked distinctively the criteria that characterise their modern counterparts as science. Was it type of mind, lack of a sense for experimental observation, lack of instruments of precision? Partly, but not altogether. Ptolemy actually measured and made a table of the refractions of

light in air and water (see Fig. 1), and he also knew from his calculations of tables of chords the values of the sines of angles ; yet he did not discover the law of Snell and Descartes. So, too, in astronomy, there was no dearth of measurements and observations, and, here even, there were genuine scientific results. Whewell says the Greeks did not have the *appropriate* ideas,—which is very true ; but we know this ; what we want to know is *why* they did not have the appropriate ideas. In point of fact the weakness of the Greeks, as contrasted with modern nations, in experimental science, is no more inexplicable than the weakness of individuals, as contrasted with one another. If Ptolemy did not discover the law of the refraction of light with all the facts and all the necessary intellectual tools in his possession, neither did Galileo discover the causes of

PTOLEMY'S TABLE OF REFRACTION. (150 A. D.)¹

ANGLES OF INCIDENCE AS MEASURED	ANGLES OF REFRACTION AS MEASURED	EXACTER VALUES
10°	8°	7°17'
20°	15.5°	14°47'
30°	22.5°	21°54'
40°	28°	28°40'
50°	35°	34°52'
60°	40.5°	40°16'

Fig. 1.

the action of atmospheric pumps, although he, too, knew all the facts. Science is a continuity ; it must not be conceived as a series of isolated products springing up in isolated brains, but as a continuous product taking shape in a succession of brains ; each age is but an aspect of its evolution ; it is not necessary that all of its growth should have taken place in any one nation or at any one time.

THE DEVELOPMENT OF MODERN ANALYSIS.

But one thing the Greeks certainly did lack which was requisite to the rapid and rigorous development of physical principles, and that was what moderns call mathematical analysis. Their

¹ From Dannemann's *Geschichte der Naturwissenschaften*, Leipsic, 1898.

strength on one side was their weakness on another. Their geometry gave them a subjective notion of harmony and necessity in the universe, but it did not furnish them the means of rapidly manipulating in their own minds, and of accurately following and reproducing, the course of that harmony and necessity. Their theories, their mental models of events, remained largely speculations, because they could not readily be submitted to quantitative verification.¹ And so their notion of scientific law had not the same objective applicability, the same demonstrability, as it has to us moderns; it remained rather a postulate of metaphysics than a truth which could be verified in individual instances.

But with the development of arithmetic, algebra, and modern analysis, a new sphere of experience was unrolled,—a sphere of experience that ran parallel to physical experience, but lay immediately at hand, and was absolutely under control. Its conquest has been step for step the conquest of physical science, and has marked step for step the establishment of the concepts of law, of causality, and of determinism in science. With the mastery of the number-continuum came concomitantly the mastery of the physical continuum, and that almost barbaric worship of mechanical law which constituted the glory of the seventeenth and eighteenth centuries. No one can have lent himself to the impressions of the literature of this period without having felt the compelling truth of this idea.

First came the Hindu system of numeration, then the invention of decimal fractions, of logarithms, the solutions of cubic and biquadratic equations, the invention of algebraic notation and symbols, the theory of exponents, algebraic geometry, the theory of series, and the calculus; steadily the precision and simplicity of these systems and engines of thought were pushed toward perfection, and daily the ease and mechanical certainty of their manipulation tested by the most varied applications; until finally the

¹ This is almost the only difference in point of real superiority of the Kant-Laplace theory over that of the ancient Greek cosmogonists; and of Kelvin's vortex-theory over Descartes's: both Laplace and Kelvin could point to precise mathematical and mechanical possibilities.

whole development reached its culmination with Leibnitz and John Bernoulli in that flower of modern mathematical thought—the notion of a function.

It is unnecessary for me to insist here on the beautiful associations which the notion of a “function” awakens in the mind of the mathematical student—the iron necessity which links the parts that constitute it, the magic flow of its consequences from the simple mechanical performance of certain well-indicated operations. All who have seen and felt in the flesh the growth of an algebraic curve, and of the myriad yet determined forms which it is capable of assuming, can appreciate the conviction of the inquirers of the seventeenth and eighteenth centuries who had achieved in this field their greatest triumphs, that here was the incarnation of natural law—the Aladdin’s lamp which they had merely to rub to see the universe in all its luxuriance of form spring up bodily before their eyes. *To press a button*, and to set in action the machinery of Nature has been the dream of the savage and the scientist alike.

DESCARTES'S APPLICATION OF MECHANICS TO GEOMETRY.

The union which Descartes effected between geometry and algebra was not purely a mathematical achievement; it was also fraught with momentous consequences for mechanics and physics. Geometry became flesh here; was transplanted from the realm of ideal form to the realm of real form; now worked from within, not from without, Nature; it was conceived to have a motional, a plastic, aspect; and no more salient illustration of the development we are considering can be found than in the opening paragraphs of Book II. of the *Géométrie* (1637). Descartes objected to the ancient division of curves into *geometrical* (those made with ruler and compasses) and *mechanical* (those requiring other instruments). The ruler and the compasses were also machines, he said, and possessed no intrinsic natural precedence over other mechanisms. The essence of geometry, he remarked, was the exactitude of the reasonings, not the simplicity of the mechanical appliances used as auxiliaries to reasoning. And just as there was no mechanism so complicated as not to have its geometrical duplicate, so there was no

geometrical construct so complicated as not to have its possible mechanical duplicate. (See Fig. 2.) Geometry and algebra are exhibited here as the counterpart of the mechanics of nature, and the mechanics of nature as the counterpart of algebra and geometry. And this conception was extended by Descartes's successors to nature in its entirety.

DESCARTES'S INSTRUMENT FOR THE GENERATION OF CURVES OF HIGHER ORDERS.

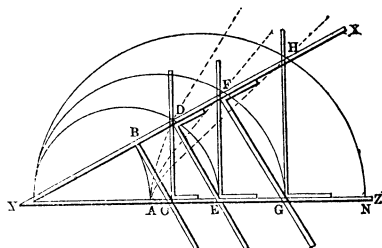


Fig. 2.

The rod YZ is stationary; the rod YX is jointed at Y so as to form a variable angle with YZ . When the instrument is closed, the points B , D , F and H meet at A . When opened, the rod BC , which is fixed on and at right angles to YX , pushes towards Z the rod CD which is movable along and at right angles to YZ ; in the same manner, CD pushes DE , which moves along at right angles to YX ; DE pushes EF ; EF , FG ; and so on *ad infinitum*. The curve traced out by the point B in this motion is a circle, while the curves traced out by the points D , F , H , etc., successively increase in complexity. But the latter curves, contends Descartes, are no less geometrical than the circle. Their mode of generation can be as distinctly and clearly conceived as that of the circle, and they are consequently just as much susceptible of mathematical treatment: the possibilities of natural forms, in other words, are duplicated by algebraical forms and are controllable by them.

THE RÔLE OF THE NOTION OF FUNCTION IN SCIENTIFIC EXPLANATION.

Having examined history, let us now analyse some of the results.

Every department of inquiry conceives its phenomena to be constructed of a certain few fundamental elements—in mechanics it is masses, spaces, and times; in biology, it is gemmules, idio-plasms, plasomes, and what nots. The scientific ideal is the reduction of the connexion of these phenomena, or of their substitutes,

the fundamental elements, to the form of a mathematical function.¹ The problem is: Given what few simple and known events, how can many unknown and complicated events be obtained by way of deduction, that is *a priori*, or without recourse to experience? The problem is equivalent to what in mathematics might be termed "the search for the *form* of the function"; the independent variables, the fundamental elements, are known or posited,—in what manner must they be handled to produce the *right* values of the dependent variables, i. e., the unknown and sought phenomena? (In the following examples, consult the corresponding ref-

TABLE ILLUSTRATING THE RÔLE OF A FUNCTION IN SCIENTIFIC EXPLANATION.

I.....	$s = \frac{1}{2}gt^2$
II.....	$T = \pi\sqrt{\frac{l}{g}}$
III.....	Roots = $F(\text{Coefficients})$
IV.....	Sensations = $\text{Log}(\text{Weights})$
V.....	Animals and Plants = $F(\text{Environment, Heredity, Selection, etc.})$
VI.....	Heredity = $F(\text{Ids, Determinants, etc.})$
VII.....	Heredity = $F(\text{Assimilation})$. . .
VIII.....	Ideas = $F(\text{Theology, Metaphysics, Science})$
IX.....	Society = $F(\text{Organisms, Mind})$.

erences of the table of typical functions.) When Galileo discovered the law that the spaces traversed by falling bodies increased as the squares of the time (I); when Huygens discovered that the times of small oscillations of pendulums were directly proportional

¹ The notion of "function" is essentially that of *dependence*. A quantity is said to be a *function* of another when its variation *depends* on that of the other. The pressure at the bottom of a vessel containing a liquid is a function of the height of the surface of the liquid; the circumference of a circle is a function of its radius. In the expressions $y = x^2 + x + 1$, $y = \sin x$, $y = \log x$, the quantity y is said to be a function of the quantity x , because there subsists between them in each case a certain relation, indicated by the symbols, rendering the values of y dependent on the values of x . The quantity y is called the *dependent* variable, and x , which is supposed to be taken at will, is called the *independent* variable. The nature of the relation subsisting between them is called the *form* of the function, and is indicated in the last two expressions by the functional symbols *sin* and *log*. When a functional relation generally is to be expressed, then certain conventional general symbols are used, as in $y = F(x)$, $y = f(x)$, $y = \phi(x)$, etc., where F , f , ϕ , etc., denote merely a general determinateness of connexion, afterwards to be more precisely specified by symbols of actual operations.

to the square root of the length and inversely as the square root of the acceleration of gravity (II); when Ferrari, Descartes, and Euler, gave their solutions of biquadratic equations (III); when in psychology Weber discovered the supposed law that sensations of pressure varied as the logarithms of the weights causing the pressure (IV); when in biology Lamarck and Darwin proclaimed the law of the variation of organisms in accordance with the actions of the environment and the principle of Natural Selection (V); when Weismann by his theory of ids and determinants (VI), and, recently, Félix Dantec, starting merely from the basis of physiological assimilation, sought to reconstruct deductively the facts of heredity (VII); when Comte stated his law of the three stages of mental development (VIII); and when in the last half century the sociologists proclaimed the analogies existing between the complex phenomena of social development and the known laws of biological and psychological development (IX); these inquirers, one and all, either actually discovered, or believed they had discovered, the *form of the function* by which, through sheer mechanical manipulation, the dependent variables of their department of inquiry could be uniquely or multiply determined from their independent variables; by which the incompletely known facts of their province could be mechanically converted into completely known facts; by which the curve of their special reality could be deductively constructed, both for the future and the past, as the graph of an algebraic equation can be plotted throughout its whole extent from a mere knowledge of certain initial numerical data.

Either there is a discoverable constant connexion between certain given events of nature, or there is none. The *form* of that connexion it is the business of the inquirer to discover. The nature of the form, its simplicity or complexity, will vary according to the character of the facts and the insight of the inquirer. But in every case the aim will be so to shape or select the initial data that the simplest possible connexion shall be forthcoming. The mathematical ideal, or rather analogy, which we have proclaimed, is indifferent; the connexion, whatever its character, whether that of the simple law of squares or cubes or of logarithmic and trigonometric

functions, or of the purest every-day analogy, is alone essential. The determination of a physical constant, the measurement of the cephalic index of the Neanderthal skull, the plotting of a curve showing the variations of the atmospheric pressure at London for the year 1898, are legitimate and necessary scientific work; they may possibly involve an increase of useful knowledge, but they are not the enunciation of a scientific law, they are not *necessarily* science; in themselves they are no more than the facts themselves; they are absolutely commensurate and coterminous with the facts; they do not involve the determination of other unknown facts. Such is the nature of a large part of the statements of sciences like natural history, meteorology, biology, psychology, etc. Many of their results are mere word-for-fact and fact-for-word descriptions of the phenomena of their respective provinces; *empirical* laws betokening nothing but what is directly expressed in them. But the correlation of the facts so discovered with indefinite numbers of other facts, not necessarily known, by virtue of the *form* of their connexion with those other facts, which connexion is the thing to be discovered, *is* the statement of a scientific law. Thus, to recur to the examples last mentioned, the statement of the reversibility of heat and energy, of the form of that reversibility, and the consequent utilisation of the physical constant involved; the deduction or induction from the measurements of the skull of the primitive Neanderthal man of a criterion for the determination of the race-characters of pre-historic humanity; the indication of a definite form of dependence of *unknown* atmospheric phenomena on the variations of the barometric curve at London;—all these are or would be the enunciations of scientific law as properly understood. We see that in all cases we return to our primitive type and ideal.

I will now give two very simple instances of actual scientific laws, and remark in connexion with them, the special features of economy of thought, prophesy of the future, and reconstruction of the facts of the past, which they involve.¹ And since we are con-

¹ Mach, *Mechanics*, pp. 147, 481 et seq.; and *Popular Scientific Lectures*, p. 186 et seq.

cerned with their formal aspects only, we shall disregard all their self-evident physical obscurities.

THE LAW OF FALLING BODIES.

For us, who from our school-days are as familiar with the laws that govern the fall of bodies as we are with the fact that the earth revolves about the sun, it is difficult to imagine the ideas which prevailed on this subject in the early years of the seventeenth century, when people were accustomed to believing rather what they were told than what they saw. Yet simple as these laws may now seem to us, they constitute one of the grandest and most enduring achievements of the human intellect, and are the beginning of all modern science. We owe their discovery to the genius of Galileo. Here science can be seen in its nascent form; here its distinctive marks most clearly perceived.

The central conception of the laws of falling bodies is, that the distances traversed by a falling stone in the 1st, 2nd, 3rd, 4th, 5th, etc., second of time are as the odd numbers, 1, 3, 5, 7, 9, and so forth; that is to say, knowing how far a stone falls in the first

GALILEO'S LAW.

1ST SEC.	2ND SEC.	3RD SEC.	4TH SEC.	5TH SEC.	6TH SEC.
I	3	5	7	9	11

General formula $s = \frac{1}{2}gt^2$.

second of time, all we have to do to find how far it falls in the second second is to multiply that distance by 3; in the third second to multiply it by 5; in the fourth second to multiply it by 7; and so on. Now, every one can calculate mentally the series of odd numbers to infinity, by simply adding 2 each time to the number that precedes, or by subtracting the squares of two successive numbers, and consequently, *supposing the conditions to remain the same*,¹ every one could calculate mentally the distances traversed by a fall-

¹ It was discovered by Newton that the acceleration of gravity varied with the distance from the centre of the earth. But this fact is unessential here. The identity of the conditions is at the basis of every scientific formulation, and constitutes an aspect of the subject not to be considered at present.

ing stone in every successive second of its flight into all eternity, without so much as resorting to experience a single time after the first.

This is the significance of a natural law: to do mentally what would otherwise have to be done physically, by arduous and sometimes absolutely impossible measurements. Note its economy; note its power of prediction; note the perfect mastery involved in its use over the unseen and sensually unseizable actions of nature. Here was the beginning of that mighty flight of the human intellect by which, three centuries afterward, with pencil and paper only, the brain of Leverrier called forth from the occult recesses of the heavens, the unknown and hitherto unseen planet Neptune. Here, too, was the beginning of that wonderful development of physical science by which in our own century a blind man, Plateau, by rightly combining the right thoughts and the right facts in his imprisoned brain, foretold and foreplanned the beautiful experiments with molecular forces which it was never given him to see.

"I can call spirits from the vasty deep," said the magician Glendower in the play; "And so can I, and so can any man," retorted Percy, "but will they *come* when you do call for them?"

But they have come at the beck of Science,—and not spirits, but planets, and worlds both living and dead, in all their real beauty and all their real truth.

* * *

Let us recapitulate the lessons of Galileo's discovery. A mathematical law, the construction of the series of odd numbers, over which we have unlimited mental power, is substituted for a series of natural phenomena over which, owing to their myriad and practically inaccessible forms, we have sensually no power. We reconstruct, remodel, and mimic in the brain, by a sort of talismanic formula, which requires but a moment of time, the past and future course of nature,—namely, all the possible distances traversed in all possible moments by a falling stone. The mind *covers* nature, so to speak.

THE LAW OF REFRACTION.

We will now turn to our second example, and observe the same factors at work.

Let a beam of yellow light, AB (Fig. 3), enter at any angle of incidence a body of water, DE , LM . The beam of light suffers, in general, what is called refraction,—a breaking or bending from its course,—and in the water follows the course BC . To each of the infinite number of possible angles of incidence, i , between the vertical BF and the horizontal BL , corresponds a definite angle of

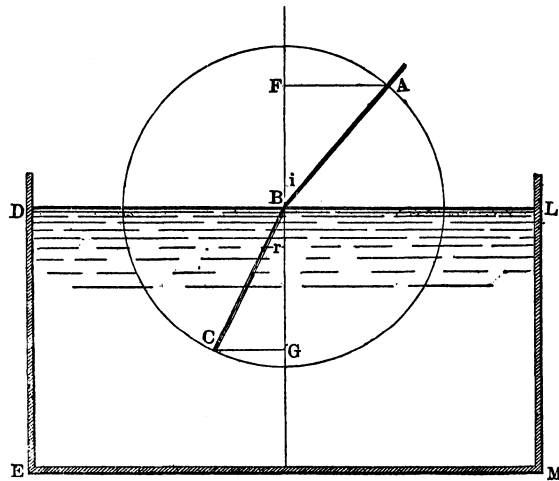


Fig. 3.

refraction, r . What is the relation that in each case connects the two? What is the magical formula by which, from knowing only the one angle, i , the unknown magnitude of the other angle, r , can be ascertained? Given what data, and given what universal rule or law, can the unknown facts, the directions of the refractions in the water, be accurately calculated, accurately remodelled in the imagination, without recourse to experience, without any actual measurement whatsoever? This is the question that Science puts. It is economic in its aspect, it is prophetic in its aspect. Thousands and thousands of possible cases, all utterly beyond the power of the human brain to remember,—how can they be reduced to a single case?

The answer was given in the formula of Descartes $\frac{\sin i}{\sin r} = n$, which says that if we divide, in any case whatever, the line FA by the line CG , the quotient will always be the same, namely a constant number, n , which in the case of water is $4/3$. To embrace, therefore, all the multitude of cases of the refraction of light from air into water, all we have to do, knowing Descartes's law, is to make a *single* measurement, once for all time. Having made that measurement, and having found the value of the constant, n , which is here $4/3$, we can, by the help of a table of sines, calculate in advance the angle of refraction of a ray of light, AB , falling on water, DL , at any angle of incidence, from 0° to 90° , that is to say, in any position which AB may take, from BF to BL . It is only necessary to divide a single number each time by $4/3$, and to find in the table the angle corresponding to the quotient,—a task of a few moments replacing with the pencil what might in some phases of modern technical industry be the labor of hours with costly instruments.

The significance of those special aspects of natural law which we have just emphasised may be brought out still more clearly by the following consideration. It is possible, by a simple combination of four rods and a pivot, to construct a working model of the laws of refraction, which, by a mere backward and forward movement, can be made to *mimic* and indicate all possible refractions. Such a machine for air and water is represented in Fig. 4. Two rods, AOK and BO , are pivoted independently at the center, O , and rest their free ends, A and B , in grooves in another rod, DC . The rod, DC , is free to move backward and forward along MN . Now, if the line, AO , measures four inches, and the line, BO , three inches, then as the rod, DC , slides back and forth, the arrow, K , will indicate mechanically every possible angle of refraction for every possible angle at which the ray, BO , can strike the surface of the water.¹

¹ This model is merely a mechanical reproduction of Snell's geometrical construction; Snell having antedated Descartes in the discovery of the law of refraction.

What have we done here? We have replaced Nature by a machine. The machine mimics Nature. By a mere mechanical movement, I can produce by the machine in miniature all the possible effects that Nature can produce in reality,—at least, in that particular aspect in which we are here studying Nature. I can follow, mimic, and outrun, Nature, without recourse to Nature. The simplicity and economy of the machine is the first thing to be noted; its power of being substituted for Nature, its being a miniature copy of Nature, is the second thing to be noted.

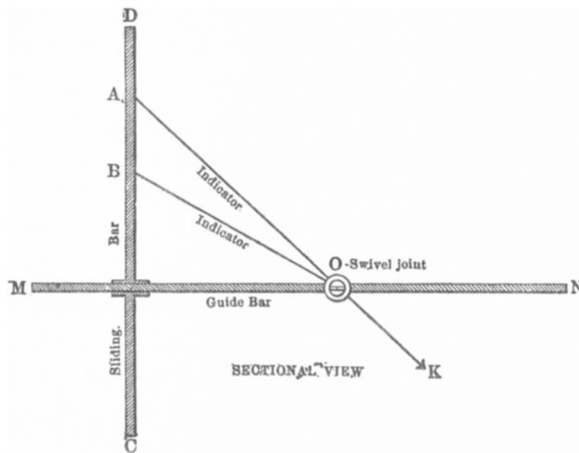


Fig. 4.

DESCRIPTION AND COMPARISON.

I shall now interpolate a few remarks as to the meaning of the term *description*, on which we seemingly threw a slight in speaking of the natural sciences. *Bare* description does not necessarily constitute the essence of science. When Kirchhoff said that it was the object of mechanics to *describe* in the simplest and completest manner possible the motions occurring in nature, he demonstrably did not mean that the essence of mechanics was a statistical word-for-fact tabulation of the motions of the universe; if he did, the *Mécanique Analytique* of Lagrange might have been written by a Patagonian savage. He merely aimed at a restoration of the Newtonian ideal of science, which had been intimated before him by D'Alem-

bert. That ideal was, in stating the facts to state *only* the facts, or relations of facts, involved, and not to state purely superfluous matter and creations of the imagination, or even analogies, not involved. It was description of actualities as opposed to description of fictions. *Hypotheses non fingo*, said Newton; "I make no hypotheses." I state only what is necessary and sufficient to *describe* what gravitation is, what moving force is; I supply no supererogatory speculations as to what are their occult causes.

The law of the inverse squares *is* a description of the motions of the heavenly bodies, but it stands on an entirely different plane from the description of a topographical chart of the State of Illinois. The real aim of Newton and Kirchhoff was the ousting of metaphysics from science. The ultimate object in building up the science of mechanics is to afford *the means* for rapidly and completely describing the motions of nature; the formulæ of mechanics potentially involve the description, but in themselves they are more than the description.

There is a deep significance in the remark that Agassiz is said to have made to the naturalists of America in his day: "You are too much *déscriptif*, too little *comparatif*." The element of comparison is at the basis of all scientific explanation; and scientific description, understood in its highest and in its lowest sense, always proceeds by comparison. When the savage saw a spirit in the magnet; when Newton likened the falling of the moon to the earth, to the phenomenon of the falling of a stone to the ground; when Pascal repeated with mercury in water the experiments which nature was daily making in the atmospheric ocean around him; when electrified bodies were conceived to be freighted with fluids which attracted and repelled each other; when the development of the foetus was conceived as a repetition of the development of the race; in each case people regarded these phenomena as explained. For unfamiliar events, familiar events were substituted, which the imagination readily followed. The analogy, or resemblance at the basis of the explanation, will vary from the crudest and wildest of resemblances to such exact comparisons as the undulatory theory of light and electricity. But it is at bottom always comparison.

The terminology of colors and numbers even was originally based on comparison. But when colors and numbers acquired significance independently of any definite objects and any definite connexion, they were raised to the abstract plane; and explanations which involve such elements have been called *direct descriptions*,¹ on the ground of the fact that the comparisons involved in them have been shorn of all concreteness and etherialised to the point of suggesting no obscuring connotations whatever. The notion of function itself has passed through this process; it was geometrical with Leibnitz; algebraical with John Bernoulli; and has become a general and almost purely logical concept with recent thinkers.

And so we may conceive the instruments of mathematics itself as analogies and models, only of an ultimate and highly abstract order; quantitatively, the course of nature runs as the course of certain functions or mathematical analogies and models runs; and thus we can bring under one conception, the whole theory of scientific explanation. *Direct* description by absolute abstract analogy is the ideal; *indirect*, by more or less crude analogy, is most commonly the practice. The former has been best exemplified in Galileo, Newton, and the French physicists of the Revolution, culminating in the theory of Boscovic that matter, the ether, and the two electric "fluids" were sums of mathematical points acted on by forces which were functions of their distances; the latter in Faraday, Maxwell, and Lord Kelvin, who spoke in parables, dealt with so-called mechanical analogies and models and dynamical illustrations, and expressly disclaimed the reality of their fictions, but revealed withal a magnificent expanse of truth. Can this be said to be a reaction?

Professor Boltzmann² seems to think so. But the reaction will be found to be apparent only. The famous model from which Maxwell is said to have derived the equations which so touched the imagination of Hertz was itself the incorporation, or at least the

¹ Mach, *Popular Scientific Lectures*, pp. 236 et seq.

² *Ueber die Methoden der theoretischen Physik*, in the *Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente*, Munich, 1892.

suggestion, of those equations ; and only in so far as it actually agreed in a formal aspect with reality was it of value to Maxwell. The *model* constituted the real discovery of Maxwell ; if he had not hit upon the right model, he would not have hit upon the right equations ; ultimately the crude model was abandoned, and the equations alone substituted for reality. A very rough diagram is a great help in solving a problem in geometry ; but the roughness of the diagram is not the gist of the solution ; the diagram merely helps to carry the thought and suggest the true relations ; only in

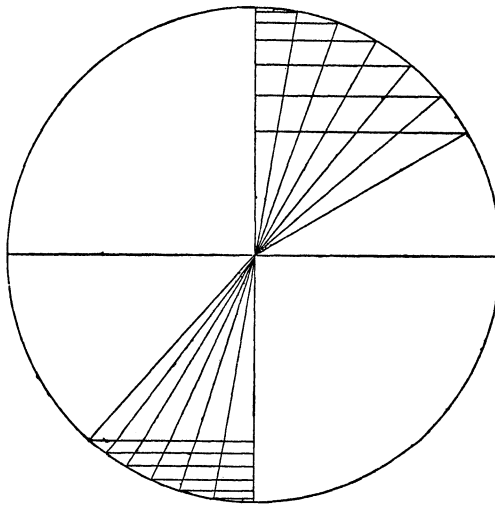


Fig. 5.

so far as the rough diagram typifies the true relations is it essential to the result.

We may very aptly illustrate this point by a comparison of Ptolemy's research on the law of refraction and the researches of Snell and Descartes. If Ptolemy, with his knowledge of the corresponding angles of incidence and refraction as exhibited in Fig. 1, had constructed a model of these varying angles, in which automatically moving strings represented the varying trigonometrical functions, as exhibited for one case in Fig. 5, he could on careful contemplation have scarcely omitted to observe the constancy of the relations between the lengths representing the sines of the two

angles. He might thus have been led ultimately to substituting for the measured lengths the numbers which he had calculated in his tables of chords, corresponding to our tables of sines, and so would have replaced his Table by a law for its construction, and have afterwards been able to dispense entirely with the rough model which had led him to his discovery. But it would have been the agreement of the model with reality that would have led him to his discovery, and not its crude or arbitrary character. The essential feature is the law which he would have deduced from the model, which may be characterised as the refinement of the model. And it is so with the mechanical models and dynamical illustrations of Maxwell. "I have used them," says Maxwell, "to assist the imagination, not to account for the phenomena." And again, "The author of this method [referring to Lord Kelvin's "Mechanical Representation of Electric, Magnetic, and Galvanic forces"] does not attempt to explain the origin of the observed forces by effects due to these strains in the elastic solid, but makes use of the mathematical analogies of the two problems *to assist the imagination* in the study of both."¹

I trust that the conception of scientific explanation, or scientific law, which makes of it a *description* of Nature, has now been clarified. We observe that it involves no more than an elimination of the metaphysical elements from Science, economy and simplicity of the notions of Science, and the power of reconstructing Nature. But we saw that there was more involved in it than a bare *recital* of experience. It involved *prediction* of experience and *reconstruction* of experience. It included that additional something which Goethe had in mind when he said, "*Experience is only half of experience.*"

We have now reached the gist of our question. Let us carry ourselves back in thought to the discussions of Galileo's law and of the law of Snell and Descartes, and interweave them with the considerations which have just preceded.

¹ Maxwell's *Scientific Papers*, Vol. I., pp. 452-453.

SCIENTIFIC SYSTEMS, MODELS, AND MACHINES.

All scientific systems, all so-called natural laws, are intellectual machines, mental working-models, or mental rules, for reproducing or reconstructing the course of Nature in the imagination. The system, or law, is substituted for Nature, it is studied in the place of Nature, and if it is a true system, a true law, the consequences of Nature can be spun off from it by sheer mechanical manipulation, as accurately and surely as wire can be spun from copper by the machines of a wire-mill. The history of astronomy affords the best example of this truth. All astronomical systems, the ancient systems of Eudoxus, Callipus, Cleanthes, and Ptolemy, no less than the modern system of Copernicus, are machines for mimicking and so reproducing for all past and future time in thought, the motions of the heavenly bodies.¹ They all did it more or less well; they were all more or less true; but none of them reached the simplicity, accuracy, and beauty, of the machinery invented by Copernicus. That machinery, with the improvements made by Kepler and Newton, has been transformed into a perfect imaginary working-model of the universe, and its study has, for the thousand and one purposes of philosophy and civilisation, been substituted for the study of that universe. Time and space have been annihilated by it. The necessity of new experience has been partly eliminated by it. The famous mathematician and astronomer, Sir William Rowan Hamilton, did not look through his telescope more than once or twice a year. He used to say, "That is my deputy's business. The stars move all night; but what interests me is the high *mathesis* that accounts for their movements."

EXPLANATION ITSELF A NATURAL PHENOMENON.

But how is this substitution of mere mental products for Nature possible? How is it possible by running the gamut of the odd numbers to foretell the spaces traversed by falling bodies; how possible by setting purely mental models a-going to mimic and

¹ See Adam Smith, *Essays on Philosophical Subjects*, London, A. Murray & Sons.

predict the course of Nature? Is Nature compelled to follow these so-called mental fictions? They do seem to exercise a constraint upon her. Nature is said to *obey* certain laws, as if these laws were beings outside of Nature and forced Nature to do their bidding. This question is the crux of the philosophy of science. Its solution has been rendered doubly difficult by our dualistic and animistic habits of language, which are the inheritance of a past reaching back to the days of savagery, and which have become bone of our bone and flesh of our flesh. I can only hint here at its solution.

Nature is as it is. It simply exists. There are no mysteries, no breaks, gaps, or disjunctions in Nature. Contradictions and inconsistencies are impossible in Nature; they are a synonym of non-existence, and exist only in the mind, which has not yet been able by the instruments at its command to mimic the actions of Nature, which has not yet adapted *its* formal mechanism to the actual mechanism of reality. To explain Nature, we have to cast about in the armory of the mind for that particular formal scheme which fits and covers the formal skeleton that is at the core of Nature; or, rather, eliminating personality, and viewing the matter objectively, the formal scheme of Nature, by its incessant action on the brain of man, *causes* the right and adequate mechanism of explanation to appear. It is an interaction between two integrant parts of Nature, not between something which is within and something which is without. The mind does not force Nature into a bed of Procrustes. It gives back to Nature merely what it takes from Nature. The mind and Nature are parts of one and the same whole, reciprocal reflexes, microcosm and macrocosm. In this view, explanation is itself a natural process, an interaction in the bosom of Nature herself, of the same kind as that interaction between undifferentiated protoplasm and rays of sunlight by which the organs of vision have been produced. In this view, dualism is eliminated, and the conformity of the mind, or so-called subjectivity, to the world, or so-called objectivity, appears as the natural conformity of two co-ordinate parts of the same consistent and orderly whole. Science takes its rise—and this is an important point—as the forms of life have taken their rise, by the adaptation of

*the mind to its environment.*¹ When the adaptation has been effected, we say a discovery has been made. We should say, the discovery has made itself.

Why the so-called creations of the brain, the working-models of events which we call scientific laws and scientific systems, run, in part of their extent at least, parallel to Nature, and may be substituted outright for Nature, is now becoming apparent. Nature, concentrated, abstracted, and abridged, is simply substituted for Nature. When we think that we are playing upon Nature, as we would upon an instrument, adapting her melodies to our scores, we are really playing an instrument (or, rather, that instrument is playing itself), which has been previously attuned by the labors of untold æons to the diapason of Nature's orchestra. The two vibrate sympathetically.

Contemplating thus the functions of purely formal thought, or the canons of logic and mathematics, in the explanation of Nature, and the marvellous power and comprehensiveness of these delicate intellectual engines, themselves the offspring of Nature, we are in some position to grasp the divine state of mind of Pythagoras, who, in discovering the parallelism of existence with the laws of number, believed that he had penetrated to the very core of Being. In some respects he had. The magnitude of his achievement has enshrouded his figure with a really divine halo. And with partial justice. Here was the beginning of an evolution which has culminated in the radiant crown of glory of modern science, and in these same realities as revealed by growing science, a modern philosophy has discerned the workings of an immanent Deity itself. We cannot wonder that as the inaugurator of this evolution tradition has credited the sage of Samos with the power of divine and occult communion with Nature. As our greatest poet has it :

"There was not the smallest orb which he beheld
But in its motion like an angel sang,
Still quiring to the young-eyed cherubim."

Such harmony was in his immortal soul.

¹ Mach, *Popular Scientific Lectures*, p. 214 et seq.

But times have changed. Through the magic wand of science, even to us "muddy vestures of decay," the harmony of the spheres has become audible music. By a long line of philosophers and scientists, beginning with the great Grecian Mystic, and continuing through Galileo, Kepler, and Newton, the "patines of bright gold" which moved the poetic heart of Shakespeare have been plucked from the "floor of heaven," and laid in all their brilliancy at our feet. This result even Pythagoras could not have foreseen.

THOMAS J. McCORMACK.

LA SALLE, ILL.